

## Appendix

### Teacher Information

This section provides general information, practice data, and answers for the student activities.

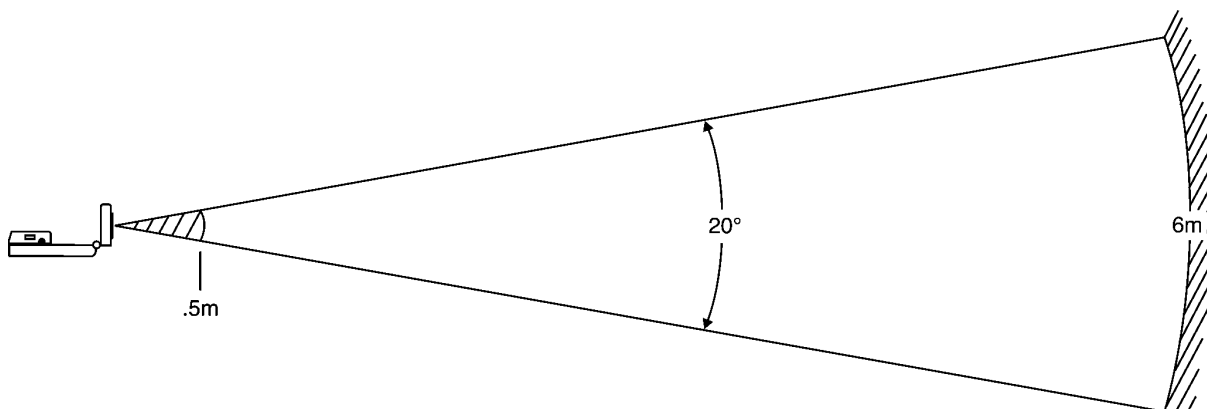
### Activity Notes

- ♦ The CBR setup instructions are included in each activity.
- ♦ If you need additional information, refer to the *Getting Started with CBR* manual included with the CBR.
- ♦ Remind students that the CBR records motion in front of it, not to the side.
- ♦ Be sure that students stay within the range of the CBR (0.5 - 6 meters).
- ♦ Most students prefer to face the CBR when walking. This allows them to stay directly in front of the unit during data collection.

### The Clear Zone

The path of the CBR beam is not a narrow, pencil-like beam, but fans out in all directions up to  $10^\circ$  in a cone-shaped beam.

To avoid interference from other objects in the vicinity, try to establish a clear zone in the path of the CBR beam. This helps ensure that objects other than the target are not recorded by the CBR. The CBR records the closest object in the clear zone.



## Activity 1: Stretching a Penny

### Data Collection

When your students perform this activity, the results will look similar to the examples below. Calculations given in the **Questions** section are based on this sample data.

Number of Pennies	Distance to the Plate (meters)
0	1.42
5	1.32
10	1.21
15	1.10
20	1.00

### Questions

1.	Number of Pennies	Stretch (meters)	Stretch (centimeters)	Stretch per Penny (centimeters/penny)
	0	0	0	0
	5	0.10	10	2.0
	10	0.21	21	2.1
	15	0.32	32	2.1
	20	0.42	42	2.1

- Answers are given in the table above.
- Answers are given in the table above.
- They are all about 2.1 centimeters per penny.
- 0.88 m or 88 cm
- 0.54 m or 54 cm
- 0.021 meters per penny or 2.1 centimeters per penny.
- $\frac{54 \text{ cm}}{2.1 \text{ cm / penny}} \approx 26 \text{ pennies}$
- 26 pennies. They are the same.
- $100 * 2.1 \text{ cm / penny} = 210 \text{ cm}$
- $\frac{0.5 \text{ m} * 100 \text{ cm / m}}{2.1 \text{ cm / penny}} \approx 24 \text{ pennies}$

### Make a Mathematical Statement

- $p = \frac{s}{2.10}$
- The force would be the weight of the pennies.
- The constant of proportionality would be  $k = \frac{1}{2.10} \text{ pennies/cm}$ .

4.  $s = 2.10 p$
5. stretch and number of pennies
6. The constant of proportionality would be  $k = 2.10 \text{ cm/penny}$ .

## Activity 2: Flipping a Penny

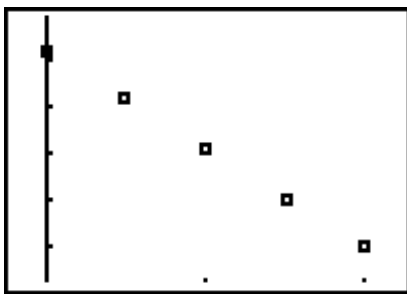
### Data Collection

When your students perform this activity, the results will look similar to the examples below. Calculations given in the **Questions** section are based on this sample data.

Number of Pennies	Distance to the Plate (meters)	Distance to the Plate (centimeters)
0	1.42	142
5	1.32	132
10	1.21	121
15	1.10	110
20	1.00	100

### Questions

1.

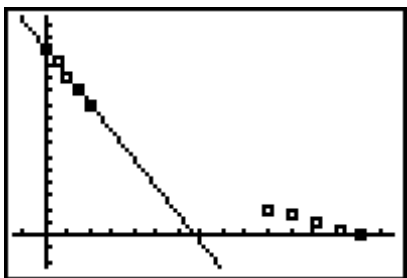


2.  $y = -2.12x + 142.2$

3. The slope is  $-2.12$ , which means that the distance between the plate and CBR decrease 2.12 centimeters per penny added.4. The  $y$ -intercept is 142.2 centimeters, which is the distance from the plate to the CBR without any pennies on the plate.

5. The line fits the points very well.

6.



7.  $y = -0.47x + 67.055$

8. The slope is  $-0.47$ , which means that if 0.47 pennies are removed, the distance from the CBR to the plate will increase by 1 centimeter.9. The  $y$ -intercept is 67.055, which means that 67.055 pennies on the plate will make the distance from the CBR to the plate 0 centimeters.

10. They both have negative slopes and intersect at a common point.

11.

P Number of Pennies	Y1(P)	Y2(Ans)
3	135.84	3.21
9	123.12	9.19
18	104.04	18.16
75	-16.80	74.95

Although entering 75 produced a negative value and is not realistic for the problem, it is realistic for inverses.

12. The last column is approximately the same as the first column.

13.  $y = x$

14.  $y = x$

15. The two graphs are the same.

16. For each value of  $x$  that is entered, the output is the same.

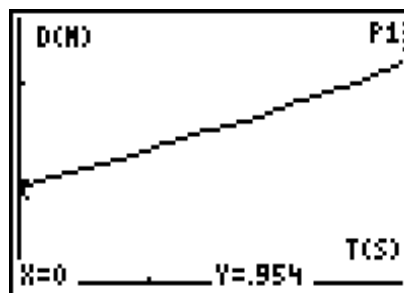
## Activity 3: Walk This Walk

### Notes

- Be sure that the CBR is pointed at the waist or chest of the walker. It may give poor results if it is aimed at the legs of the walker. Also, clear desks and chairs away from the path of the walker so that the ranger does not reflect from those objects rather than the walker.
- You may want to discuss linear equations and the concept of slope and y-intercept with the students.

### Instructions

1. No answer required.
2. No answer required.
3.
  - a. The graph shows an increase with time if the walker moves away from the CBR.
  - b. The graph shows a decrease with time if the walker moves towards the CBR.
  - c. The graph show a horizontal line if the walker is stationary.
4. The graph shows a line with positive slope for constant speed away from the CBR.



### Data Collection

When your students perform this activity, the results will look similar to the examples below.

L1	L2
0	0.954
0.03185	0.965
0.12738	1.003
0.15923	1.020
0.25477	1.056
0.35031	1.089
0.44584	1.127
0.54138	1.163
0.63692	1.183
0.73246	1.215
0.82800	1.247

## Questions

1. The plot is shown in #5 on the previous page.

<b>t1=</b>	0	<b>d1=</b>	0.954
<b>t2=</b>	2.101	<b>d2=</b>	1.798

$$\Delta d = 0.844$$

$$\Delta t = 2.101$$

$$m = 0.402 \text{ meters per second}$$

2.  $b = 0.954$
3.  $Y = 0.402X + 0.954$
4. Answers will vary.
5. The slope represents the speed or velocity.
6. The  $y$ -intercept represents the starting position.
7. Equation:  $y = x + 1$ .
8. The graph would show a negative slope.

**9 through 13.**

The sample calculations are not repeated here. The slope of the equation in this section is negative.

14. Any equation of a person moving away from the CBR at a constant speed is linear with positive slope. The equation for the motion of a person walking towards the CBR at a constant rate is linear with a negative slope.
15. A person moving with negative velocity is moving towards the CBR.

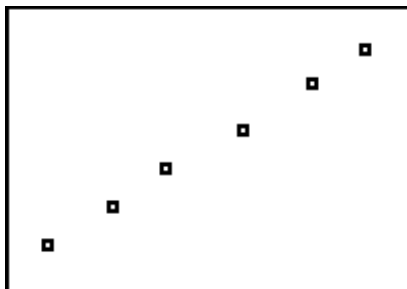
## Activity 4: What is Gravity?

### Data Collection

5. A sample of collected data is shown below. The angles were all small (less than  $10^\circ$ ) in each trial. Enter the  $\sin(\theta)$  in **L1** and the acceleration in **L2**.

Trial	d	h	$(t_1, v_1)$	$(t_2, v_2)$	$\sin(\theta)$ (L1)	acceleration (L2)
1	1.5 m	0.073 m	(0.53, 0.33)	(1.29, 0.68)	0.049	0.461
2	1.5 m	0.095 m	(0.47, 0.40)	(1.31, 0.90)	0.063	0.595
3	1.5 m	0.112 m	(0.21, 0.31)	(0.51, 0.52)	0.075	0.700
4	1.5 m	0.134 m	(0.21, 0.31)	(0.64, 0.68)	0.089	0.860
5	1.5 m	0.157 m	(0.57, 0.61)	(0.95, 0.99)	0.105	1.000
6	1.5 m	0.175 m	(0.30, 0.49)	(0.85, 1.10)	0.117	1.109

13.



### Questions

Answers are given based on the examples above.

1.  $y = 9.615x - .011$

2.

Angle	$\sin(\theta)$	acceleration
$30^\circ$	0.500	4.797
$45^\circ$	0.707	6.787
$60^\circ$	0.866	8.316
$75^\circ$	0.966	9.277
$85^\circ$	0.996	9.566

3. 9.615

4. 9.615

5. The absolute error is  $9.8 - 9.615 = 0.185$ . The relative error is  $\frac{.185}{9.8} \times 100 = 1.9\%$ .

6. It slows the particle down.

7. Negative, since the velocity is in the opposite direction as that of the acceleration.



8. It causes the object to speed up.
9. Negative, since the velocity is in the same direction as that of the acceleration.

## Activity 5: How Far Did You Walk?

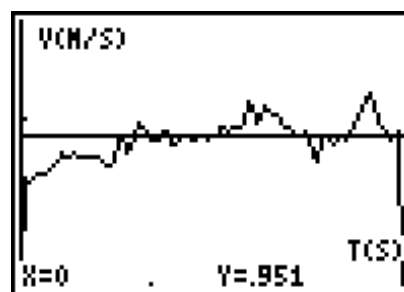
### Data Collection

When your students perform this activity, the results will look similar to the practice data below and in #2. If you would like to practice this activity without collecting the data, use the list editor to enter the time in **L1**, the distance in **L2**, and the velocity in **L3**.

Time (L1)	Dist (L2)	Vel (L3)
0	0.80	0.39
0.03	0.83	0.83
0.32	1.07	0.90
0.51	1.24	0.89
0.70	1.41	0.86
0.89	1.58	0.97
1.02	1.71	0.94
1.18	1.86	0.92
1.34	2.01	0.94
1.50	2.16	0.96

To view a plot, press **2nd** **[STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **ENTER**. Select  $\text{L1}$  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **ZOOM** and select **9:ZoomStat**.

- $v = 0.95$
- The jerky motion of a person walking causes the imperfections in the graph.



### Questions - Part One

- It is 0.
- The acceleration is 0 for constant velocity. They are the same, 0.

Time (sec)	Velocity	Area
0.5	0.95	0.48
1.0	0.95	0.95
1.5	0.95	1.43
2.0	0.95	1.90
2.5	0.95	2.38
3.0	0.95	2.85

- $d = .8 \text{ m}$

5.

Time (sec)	Distance from the wall	Distance Traveled
0.5	1.24	0.44
1.0	1.71	0.91
1.5	2.16	1.36
2.0	2.67	1.87
2.5	3.15	2.35
3.0	3.59	2.79

6. See **Distance Traveled** in the table above.
7. They are all within 0.03 to 0.07 meters of each other.
8.  $d = .95 * t$

## Questions - Part Two

To enter the program, press **PRGM** **▶** **▶** and select **1:Create New** to create a new program.  
 To enter a name type **[A] [R] [E] [A]** and press **ENTER** to move to the first line of the program.  
 Then enter the following commands.

Program Line	Keystrokes
ClrHome	<b>PRGM</b> <b>▶</b> <b>8</b> <b>ENTER</b>
ClrDraw	<b>2nd</b> <b>[DRAW]</b> <b>1</b> <b>ENTER</b>
$\emptyset \rightarrow T$	<b>0</b> <b>STO▶</b> <b>ALPHA</b> <b>[T]</b> <b>ENTER</b>
Prompt N	<b>PRGM</b> <b>▶</b> <b>2</b> <b>ALPHA</b> <b>[N]</b> <b>ENTER</b>
int(94/N)→S	<b>For the TI-83:</b> <b>MATH</b> <b>▶</b> <b>5</b> <b>9</b> <b>4</b> <b>÷</b> <b>ALPHA</b> <b>[N]</b> <b>)</b> <b>STO▶</b> <b>ALPHA</b> <b>[S]</b> <b>ENTER</b> <b>For the TI-82:</b> <b>MATH</b> <b>▶</b> <b>4</b> <b>(</b> <b>9</b> <b>4</b> <b>÷</b> <b>ALPHA</b> <b>[N]</b> <b>)</b> <b>STO▶</b> <b>ALPHA</b> <b>[S]</b> <b>ENTER</b>
For(I,1,94-S,S)	<b>PRGM</b> <b>4</b> <b>ALPHA</b> <b>[I]</b> <b>,</b> <b>1</b> <b>,</b> <b>9</b> <b>4</b> <b>-</b> <b>ALPHA</b> <b>[S]</b> <b>,</b> <b>ALPHA</b> <b>[S]</b> <b>)</b> <b>ENTER</b>
$L_1(I) \rightarrow A$	<b>2nd</b> <b>1</b> <b>(</b> <b>ALPHA</b> <b>[I]</b> <b>)</b> <b>STO▶</b> <b>ALPHA</b> <b>[A]</b> <b>ENTER</b>
$L_3(I+.5S) \rightarrow B$	<b>2nd</b> <b>3</b> <b>(</b> <b>ALPHA</b> <b>[I]</b> <b>+</b> <b>.</b> <b>5</b> <b>ALPHA</b> <b>[S]</b> <b>)</b> <b>STO▶</b> <b>ALPHA</b> <b>[B]</b> <b>ENTER</b>
$L_1(I+S) \rightarrow C$	<b>2nd</b> <b>1</b> <b>(</b> <b>ALPHA</b> <b>[I]</b> <b>+</b> <b>ALPHA</b> <b>[S]</b> <b>)</b> <b>STO▶</b> <b>ALPHA</b> <b>[C]</b> <b>ENTER</b>
Line(A, $\emptyset$ ,A,B)	<b>2nd</b> <b>[DRAW]</b> <b>2</b> <b>ALPHA</b> <b>[A]</b> <b>,</b> <b>0</b> <b>,</b> <b>ALPHA</b> <b>[A]</b> <b>,</b> <b>ALPHA</b> <b>[B]</b> <b>)</b> <b>ENTER</b>
Line(A,B,C,B)	<b>2nd</b> <b>[DRAW]</b> <b>2</b> <b>ALPHA</b> <b>[A]</b> <b>,</b> <b>ALPHA</b> <b>[B]</b> <b>,</b> <b>ALPHA</b> <b>[C]</b> <b>,</b> <b>ALPHA</b> <b>[B]</b> <b>)</b> <b>ENTER</b>
Line(C, $\emptyset$ ,C,B)	<b>2nd</b> <b>[DRAW]</b> <b>2</b> <b>ALPHA</b> <b>[C]</b> <b>,</b> <b>0</b> <b>,</b> <b>ALPHA</b> <b>[C]</b> <b>,</b> <b>ALPHA</b> <b>[B]</b> <b>)</b> <b>ENTER</b>
$S*(L_1(2)-L_1(1))*B+T \rightarrow T$	<b>ALPHA</b> <b>[S]</b> <b>×</b> <b>(</b> <b>2nd</b> <b>1</b> <b>(</b> <b>2</b> <b>)</b> <b>-</b> <b>2nd</b> <b>1</b> <b>(</b> <b>1</b> <b>)</b> <b>)</b> <b>×</b> <b>ALPHA</b> <b>[B]</b> <b>+</b> <b>ALPHA</b> <b>[T]</b> <b>STO▶</b> <b>ALPHA</b> <b>[T]</b> <b>ENTER</b>
Text(1,1 $\emptyset$ ,"AREA=")	<b>2nd</b> <b>[DRAW]</b> <b>0</b> <b>1</b> <b>,</b> <b>1</b> <b>0</b> <b>,</b> <b>2nd</b> <b>[A-LOCK]</b> <b>["]</b> <b>[A]</b> <b>[R]</b> <b>[E]</b> <b>[A]</b> <b>2nd</b> <b>[TEST]</b> <b>1</b> <b>ALPHA</b> <b>["]</b> <b>)</b> <b>ENTER</b>
Text(1,3 $\emptyset$ ,T)	<b>2nd</b> <b>[DRAW]</b> <b>0</b> <b>1</b> <b>,</b> <b>3</b> <b>0</b> <b>,</b> <b>ALPHA</b> <b>[T]</b> <b>)</b> <b>ENTER</b>
End	<b>PRGM</b> <b>7</b>

Press **2nd** **[QUIT]** to exit the program.

**Note:** You can also load the program using TI-GRAPHLINK™.

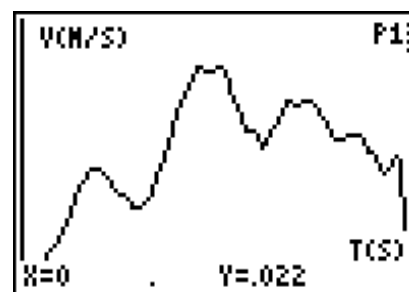
1.  $d_i = 0.8$  m
2.  $d_f = 1.95$  m

3. Sample data is given below. If you would like to practice this activity without collecting the data, enter the time in **L1**, the distance in **L2** and the velocity in **L3**.

Time (L1)	Dist (L2)	Vel (L3)
0	0.61	0.02
0.29	0.64	0.16
0.51	0.70	0.36
0.57	0.72	0.37
0.64	0.74	0.35
0.83	0.80	0.27
0.89	0.82	0.25
0.99	0.85	0.27
1.40	1.05	0.66
1.53	1.13	0.67

Time (L1)	Dist (L2)	Vel (L3)
1.72	1.25	0.48
1.78	1.28	0.47
1.85	1.30	0.43
2.07	1.42	0.57
2.23	1.51	0.57
2.42	1.60	0.46
2.61	1.69	0.46
2.80	1.77	0.36
2.90	1.81	0.40
2.96	1.83	0.19

To view a plot, press **2nd** **[STAT PLOT]**. The graph should be similar to the one shown



4. The program will *not* work with the data set given in #3 since it is designed to work with the full set of data points.

N	Area
5	1.20
9	1.18
15	1.17
23	1.20
47	1.20

5. Yes. It seems to be 1.20.
6. Total distance is  $1.95 \text{ m} - 0.80 \text{ m} = 1.15 \text{ m}$ .
7. They are within 0.05 m of each other.
8. They are the same.

## Activity 6: Intersection

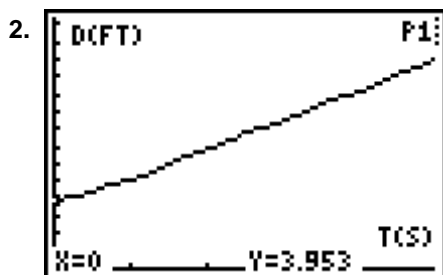
### Data Collection

When your students perform this activity, the results will look similar to the practice data below. Calculations in the **Question** section are based upon the practice data. If you would like to practice this activity without collecting the data, you can enter the time in **L1**, the distance from the wall of Walker A should be entered in **L2**, and the distance Walker B is from the wall in **L3**. To enter the data, press **[STAT]** **[ENTER]**.

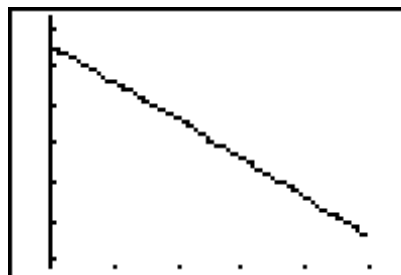
Time (L1)	Distance A (L2)	Distance B (L3)
0	3.95	15.45
1.01	5.10	14.56
2.02	6.83	13.66
3.13	8.61	12.61
4.30	10.78	11.27
4.92	11.91	10.65

To see the plot of Walker A's Distance-Time graph, press **[2nd]** **[STAT PLOT]**. Select **1:Plot1**. Press **[ENTER]**, highlight **On**, and press **[ENTER]**. Select  $\text{L1}$  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist**, and the **dot** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**.

To see the plot of Walker B's Distance-Time graph, press **[2nd]** **[STAT PLOT]**. Select **4:PlotsOff**. Press **[2nd]** **[STAT PLOT]**. Select **2:Plot2**. Press **[ENTER]**, highlight **On**, and press **[ENTER]**. Select  $\text{L1}$  for the **Type** of plot, **L1** for the **Xlist**, **L3** for the **Ylist**, and the **dot** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**.



Walker A



Walker B

**Note:** The windows will be different for Walker A and Walker B.

### Questions - Part One

1.

	Point 1		Point 2	
	Time	Distance	Time	Distance
Walker A	0	3.95	4.30	10.78
Walker B	0	15.45	4.30	11.27

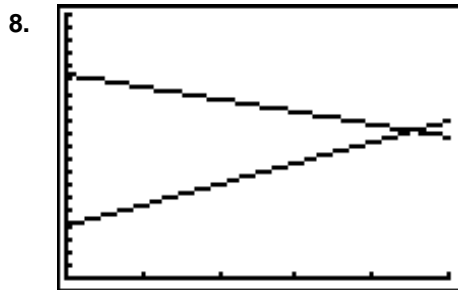
2.

	$\Delta y = y_2 - y_1$	$\Delta t = t_2 - t_1$	velocity = $\frac{\Delta y}{\Delta t} = \frac{y_2 - y_1}{t_2 - t_1}$
Walker A	$10.78 - 3.95 = 6.83$	$4.30 - 0 = 4.30$	$6.83 / 4.30 = 1.59$
Walker B	$11.27 - 15.45 = -4.18$	$4.30 - 0 = 4.30$	$-4.18 / 4.30 = -0.97$

3. The units associated with this velocity are feet per second.
4. The speed or velocity of the walker will influence the rate of change of the distance traveled by each walker.

5.	$m$	$b$	equation of line
Walker A	1.59	3.95	$y = 1.59x + 3.95$
Walker B	-0.97	15.45	$y = -0.97x + 15.45$

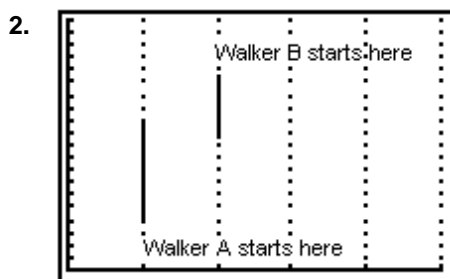
6. The  $y$ -intercept is the distance that each walker was from the wall at time 0.
7. The opposite slopes indicate that the walkers are moving in opposite directions. The positive slope of Walker A indicates that the walker is moving away from the wall while the negative slope of Walker B indicates that the walker is moving towards the wall.



9. Point of intersection: (4.49, 11.09)
10. The  $y$ -coordinate of the point of intersection gives the distance that the walkers were from the wall when they met.
11. The  $x$ -coordinate of the point of intersection gives the time that the walkers were the same distance from the wall when they met.

### Questions - Part Two

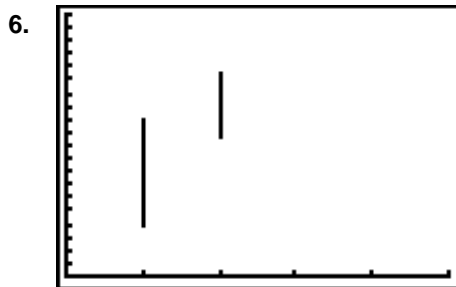
1. The graph on the calculator shows the position of the walker versus the time. It does not show the actual motion of the walker.



Wall where CBR was pointed

3.	Lane	distance to the wall
Walker A	1	$1.59t + 3.95$
Walker B	2	$-0.97t + 15.45$

4. Position of Walker A:  $1.59(5) + 3.95 = 11.9$  feet  
Position of Walker B:  $-0.97(5) + 15.45 = 10.60$  feet
5. No answer required.



7. The two graphs should be the same.

8.

Time	Distance	
	Walker A	Walker B
0	3.95	15.45
1	5.54	14.48
2	7.13	13.51
3	8.72	12.54
4	10.31	11.57
5	11.9	10.60

9. Velocity of Walker A:  $11.9 - 3.95 = 7.95$  feet  $\rightarrow \frac{7.95 \text{ feet}}{5 \text{ sec}} = 1.59 \text{ ft/sec}$

Velocity of Walker B:  $10.60 - 15.45 = -4.85$  feet  $\rightarrow \frac{-4.85 \text{ feet}}{5 \text{ sec}} = -0.97 \text{ ft/sec}$

10. They should be the same.



## Activity 7: Match This

### Notes


- This activity works well when students use the ViewScreen™ calculator and an overhead projector so that they can easily see the graph that they are trying to match. It will also work in small groups if one member of the group holds the calculator so that the walker can see the graph as he or she walks.
- Students should have some knowledge of how to write the equation of a linear function. Students should also have some knowledge of graphing piecewise functions and the use of the **TEST** and **LOGIC** menus.
- Although the students are matching a given graph, their data points will vary slightly from the given graph. For this reason, they are asked to match the equation to the collected data points rather than the given graph.

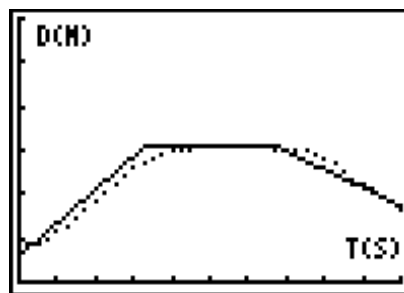
### Data Collection

When your students perform this activity, the results will look similar to the practice data below. If you would like to practice this activity without collecting the data, you can enter the following data points into the lists **L1** and **L2** through the list editor by pressing **[STAT]** **[ENTER]**.

L1	L2
0	0.770631
0.249958	0.885296
0.499916	0.974374
0.749874	1.116740
0.999832	1.259210
1.249790	1.456960
1.499748	1.650170
1.749706	1.870230
1.999664	2.035430
2.249622	2.260530
2.499580	2.407880
2.749538	2.568650
2.999496	2.695340
3.249454	2.768430
3.499412	2.897260
3.749370	2.967900
3.999328	3.010750
4.249286	3.033120
4.499244	3.079640
4.749202	3.083890

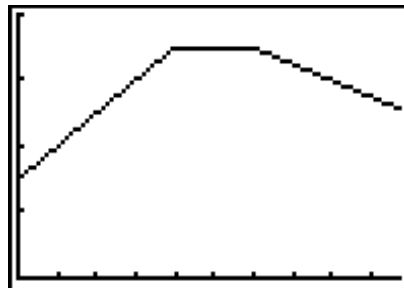
L1	L2
4.999160	3.077220
5.249118	3.069770
5.499076	3.058550
5.749034	3.055250
5.998992	3.055730
6.248950	3.056020
6.498908	3.048750
6.748866	3.033840
6.998824	3.013230
7.248782	2.989020
7.498740	2.920610
7.748698	2.807130
7.998656	2.673180
8.248614	2.539300
8.498572	2.366100
8.748530	2.240940
8.998488	2.116140
9.248446	1.944410
9.498404	1.815070
9.748362	1.689990

To view a plot, press **[2nd] [STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.



### Questions

1. **X1** = 0 and **Y1** = 0.77063; **X2** = 3.75 and **Y2** = 2.97.
2. slope equation =  $\frac{y_2 - y_1}{x_2 - x_1}$ ,  $m = 0.587$
3.  $b = 0.77$
4.  $Y = 0.587 X + 0.77$
5. **X3** = 7.25 and **Y3** = 2.99
6.  $m = 0.006$  meters per seconde, which is approximately zero.  
 $b = 3.05$  approximately
7.  $Y = 3.05$
8. **X4** = 9.75 and **Y4** = 1.69
9.  $m = -0.52$  meters per second
10.  $Y = -0.52 X + 6.76$
11. No answer required.
12. No answer required.
13. The slopes represent the velocity for that section of the walk.
14. The  $y$ -intercept represents the starting position. The strips of tape on the floor help gauge the starting point from the CBR.
15. The graph would be shifted upward by a unit of 1.
16. The  $y$ -intercept in the second section represents the constant distance since all  $y$  values are approximately equal here. The  $y$ -intercept for the third section does not have physical significance unless the student sees it as the position from which he or she would begin in order to match the last section of the graph. (This assumes that the student walked at a constant speed the entire time.)
17. See the graph shown at right.



## Activity 8: The Bouncing Ball

### Notes

- Avoid using a soft or felt-covered ball such as a tennis ball, since pulses from the CBR tend to be absorbed by these surfaces.
- If you have trouble obtaining good results, try using a larger ball.
- For best results, hold the sides of the ball and then quickly move your hands outward to release the ball.

### Data Collection


When your students perform this activity, the results will look similar to the practice data below. Calculations in the Questions section are based on the sample data below.

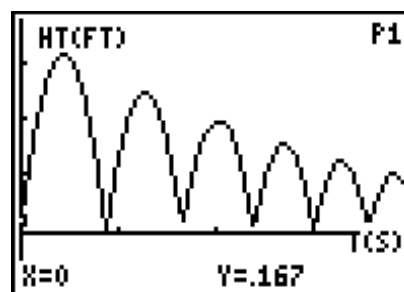
If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1**, **L2**, and **L3** and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, the distance in **L2** and velocity in **L3**.

In the practice data, the distance values are in feet and the velocity values are in feet per second.

Time (L1)	Dist (L2)	Vel(L3)
0.00	0.17	6.47
0.04	0.71	12.55
0.17	2.03	8.38
0.34	3.00	2.98
0.42	3.14	0.41
0.50	3.07	-2.19
0.71	1.90	-8.81
0.84	0.49	-12.35
0.88	0.00	0.05
0.92	0.50	11.35
1.01	1.35	8.58
1.18	2.31	3.08
1.26	2.46	0.44
1.34	2.38	-2.20
1.55	1.22	-9.24
1.60	0.79	-0.44
1.68	0.17	2.86
1.72	0.58	9.44
1.93	1.86	2.54

Time (L1)	Dist (L2)	Vel(L3)
2.02	1.96	-0.09
2.10	1.84	-2.82
2.31	0.54	-9.33
2.35	0.13	-2.81
2.43	0.64	7.46
2.60	1.48	2.37
2.69	1.56	-0.39
2.77	1.43	-3.01
2.94	0.45	-8.28
2.98	0.08	-1.49
3.02	0.32	6.15
3.19	1.16	2.46
3.27	1.25	-0.25
3.36	1.12	-2.96
3.48	0.50	-6.71
3.53	0.20	-4.03
3.61	0.46	5.90
3.74	0.94	2.10
3.82	1.02	-0.54

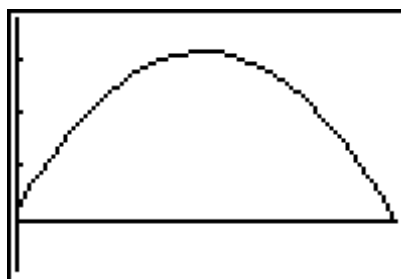
To view a plot, press **[2nd] [STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.



1.

	$t$	$y$
Xmin	0	0.17
Vertex	0.42	3.14
Xmax	0.88	0

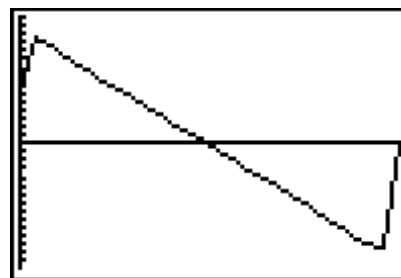
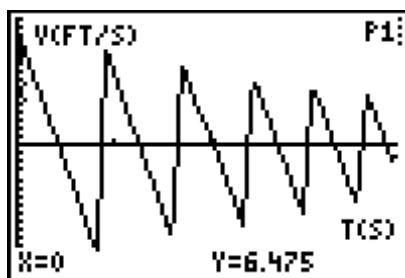
3. The graph using the sample data will not have the detail shown since fewer points were plotted.



### Questions - Part One

- No answer required.
- $A = -16$
- Equation for this parabola:  $y = -16(x - 0.42)^2 + 3.14$
- No answer required.
- Velocity of the ball bounce.

Velocity of the first bounce.



- Point 1: (0.04, 12.55) Point 2: (0.34, 2.98)  
 $m = -31.9$        $b = 13.83$        $y = -32x + 14$
- The linear regression does not work as well here since the data at the ends of the graph are not actually associated with this bounce's velocity.

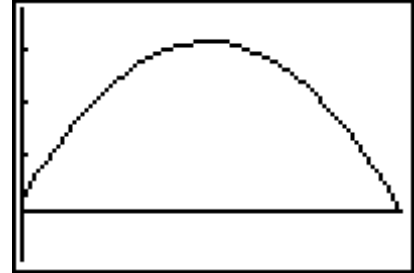
**Questions - Part Two**

1. No answer required.

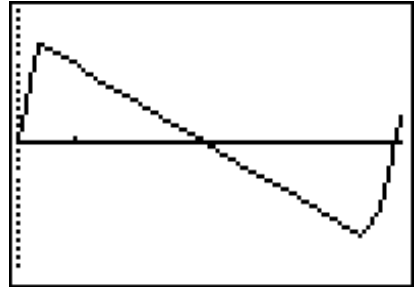
2.

	$t$	$y$
Xmin	0.88	0
Vertex	1.26	2.46
Xmax	1.68	0.17

3. The graph using the sample data will not have the detail shown since fewer points were plotted.



4.  $A = -16$
5. Equation for this parabola:  $y = -16(x - 1.26)^2 + 2.46$
6. No answer required.
7. Velocity of the second bounce.



8. Point 1: (0.92, 11.35) Point 2: (1.18, 3.08)
9.  $m = -31.81$        $b = 40.61$        $y = -32x + 41$
10. The line should fit the data very well.

**Make a Mathematical Statement**

1. The equations found in this lab are

Bounce	Distance -Time	Velocity-Time
1	$y = -16(x - 0.42)^2 + 3.14$	$y = -32x + 14$
2	$y = -16(x - 1.26)^2 + 2.46$	$y = -32x + 41$

2. Gravity influences the change in the ball's velocity.
3. In both distance equations,  $A = -16$ . In both velocity equations,  $m = -32$ .  $A$  is half of the slope of the velocity equation.
4. The only force acting on the ball is gravity. Gravity, in this problem, is  $-32 \text{ ft/s}^2$ .

## Activity 9: Conserving Energy

### Notes

- Be sure that students do not allow the cart to come closer to the CBR than 0.5 meter (the tape mark).
- It is important to provide a well-defined target for the CBR's beam. If the data plot is not satisfactory, check the cart's target surface. If it is small, you may need to attach a small piece of cardboard to the front end of the cart.

### Data Collection

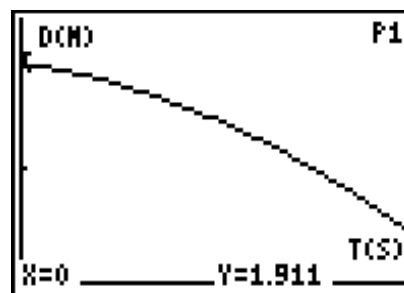
When your students perform this activity, the results will look similar to the practice data below. Calculations in the **Questions** section are based on the sample data below.

If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1**, **L2**, and **L3** and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, the distance in **L2** and velocity in **L3**.

Time (L1)	Dist (L2)	Vel (L3)
0	1.91	-0.25
0.19	1.86	-0.32
0.38	1.79	-0.44
0.57	1.69	-0.53
0.76	1.58	-0.63
0.95	1.46	-0.72
1.06	1.38	-0.77

Time (L1)	Dist (L2)	Vel (L3)
1.14	1.31	-0.81
1.34	1.14	-0.89
1.53	0.97	-0.99
1.72	0.77	-1.08
1.81	0.67	-1.14
1.91	0.56	-1.19
1.97	0.49	-1.21

To view a plot, press **[2nd]** **[STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  $\wedge$  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.



4.  $m = .492 \text{ kg}$

### Questions

- $y = -0.24x^2 - 0.26x + 1.91$  (For the practice data above, the result will be 1.92.)
- $s_0 = 1.91$     $v_0 = -0.26$     $-\frac{1}{2}a = -0.24$     $a = 0.48$
- $y = -0.49x - 0.25$
- $v_0 = -0.25$     $-a = -0.49$     $a = 0.49$
- They are both within 0.01 of each other.
- No answer required.
- $h = s \sin(\theta)$

8.  $U = mgssin(\theta)$
9.  $U = mas$
10. No answer required.
11. No answer required.
12. No answer required.
13. The graph shown is for a full set of data.



14.  $U + K = 0.484$

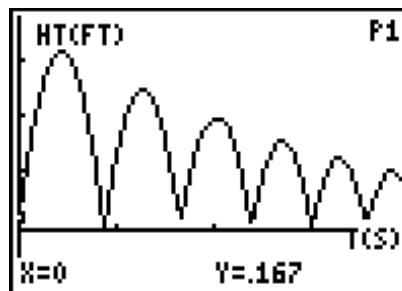
**Make a Mathematical Statement**

1.  $h(t) = \left(s_0 + v_0 t - \frac{1}{2}at^2\right)\sin(\theta)$
2.  $U(t) = mg\left(s_0 + v_0 t - \frac{1}{2}at^2\right)\sin(\theta)$
3.  $U(t) = ma\left(s_0 + v_0 t - \frac{1}{2}at^2\right) = mas_0 + mav_0 t - \frac{1}{2}ma^2t^2$
4.  $K(t) = \frac{1}{2}m(v_0 - at)^2 = \frac{1}{2}m(v_0^2 - 2av_0 t + a^2t^2) = \frac{1}{2}mv_0^2 - mav_0 t + \frac{1}{2}ma^2t^2$
5.  $U(t) + K(t) = mas_0 + \frac{1}{2}mv_0^2$
6.  $U + K = 0.492 * 0.48 * 1.92 + \frac{1}{2} * 0.492 * (-0.26)^2 = 0.470$
7. They are within 0.014 of each other.

## Activity 10: Velocity and the Bouncing Ball

### Instructions

- Sample ball bounce graph for the original exercise. For this example, we used feet instead of meters. The sample data given below is for the first bounce of this graph.



### Data Collection

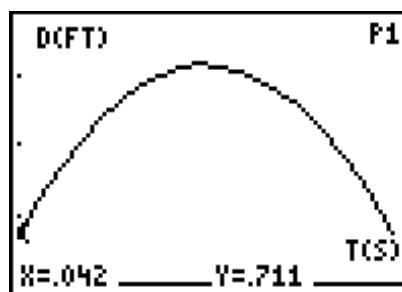
When your students perform this activity, the results will look similar to the practice data below. Calculations in the **Questions** section are based on the sample data below.

If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1**, **L2**, and **L3** and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, the distance in **L2** and velocity in **L3**. The data for one ball bounce is given below.

Time (L1)	Dist (L2)	Vel (L3)
0.042	0.711	12.547
0.084	1.221	11.244
0.126	1.655	9.677
0.168	2.033	8.379
0.210	2.359	7.070
0.252	2.627	5.794
0.294	2.845	4.426
0.336	2.999	2.983
0.378	3.095	1.684
0.420	3.140	0.413

Time (L1)	Dist (L2)	Vel (L3)
0.462	3.13	-0.880
0.504	3.066	-2.189
0.546	2.946	-3.610
0.588	2.763	-4.941
0.630	2.531	-6.201
0.672	2.242	-7.483
0.714	1.903	-8.814
0.756	1.502	-10.310
0.798	1.037	-12.016
0.840	0.493	-12.354

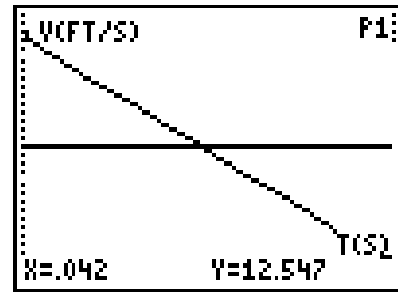
To view a plot, press **[2nd]** **[STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  $\text{line}$  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.



- No answer required.
- $y = -15.90x^2 + 13.76x + 0.17$



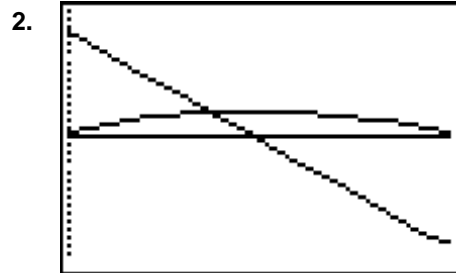
4. The function models the position data very well.



5. No answer required.
6. No answer required.
7.  $y = -31.71x + 13.76$
8. The equation models the velocity data very well.

### Questions

1. No answer required.



3.

Tme	Height	Velocity
0.042	0.711	12.547
0.126	1.655	9.677
0.210	2.359	7.070
0.294	2.845	4.426
0.378	3.095	1.684
0.462	3.130	-0.880
0.546	2.946	-3.610
0.630	2.531	-6.201
0.714	1.903	-8.814

- a. As the ball approaches the maximum height of this bounce, the velocity is positive but decreasing.
- b. As the ball falls from the maximum height of this bounce, the velocity is negative but increasing.
- c. The ball is at its maximum height for this bounce when the velocity is zero.
4.  $t = \frac{-13.76}{-31.71} = 0.43 \text{ s}$
5.  $s = -15.90(0.43)^2 + 13.76(0.43) + 0.17 = 3.15 \text{ ft}$

6. In the table in question 3, the velocity is closest to 0 at  $t = 0.462$  s. This is within 0.032 seconds of the time value found in question 5, and the height in the table is within 0.02 feet of the value found in question 5.
7.  $y = 0.17 + 13.76x - 15.90x^2$   
 $s_0 = 0.17 \quad v_0 = 13.76 \quad \frac{1}{2}a = -15.90 \quad a = -31.80$
8.  $y = 13.76 - 31.71x$   
 $v_0 = 13.76 \quad a = -31.71$
9. The initial velocities are the same. The accelerations are within 0.09 of each other.

## Activity 11: Bounce Back


### Instructions

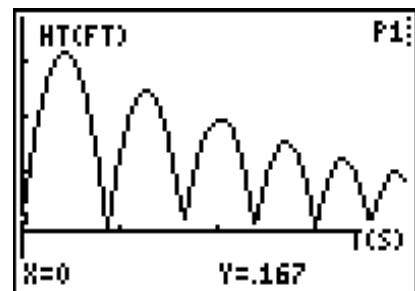
When your students perform this activity, the results will look similar to the practice data below. Calculations in the Questions section are based on the sample data below.

If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1**, **L2**, and **L3** and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, the distance in **L2** and velocity in **L3**. In this example, distances are given in feet rather than meters.

Time (L1)	Dist (L2)
0.00	0.17
0.04	0.71
0.17	2.03
0.34	3.00
0.42	3.14
0.50	3.07
0.71	1.90
0.84	0.49
0.88	0.00
0.92	0.50
1.01	1.35
1.18	2.31
1.26	2.46
1.34	2.38
1.55	1.22
1.60	0.79
1.68	0.17
1.72	0.58
1.93	1.86

Time (L1)	Dist (L2)
2.02	1.96
2.10	1.84
2.31	0.54
2.35	0.13
2.43	0.64
2.60	1.48
2.69	1.56
2.77	1.43
2.94	0.45
2.98	0.08
3.02	0.32
3.19	1.16
3.27	1.25
3.36	1.12
3.48	0.50
3.53	0.20
3.61	0.46
3.74	0.94
3.82	1.02

To view a plot, press **[2nd]** **[STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.



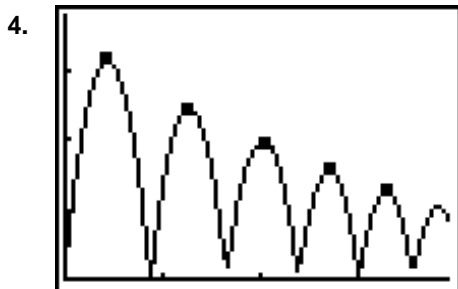
## Data Collection

1. 

Bounce	$t$	$y$
1	0.42	3.14
2	1.26	2.46
3	2.02	1.96
4	2.69	1.56
5	3.27	1.25

2. No answer required.

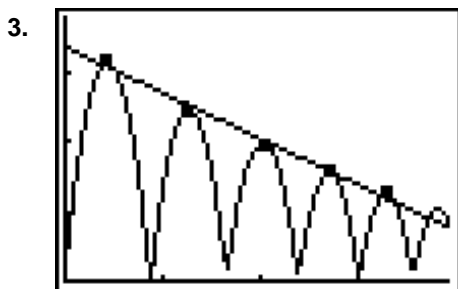
3. No answer required.



## Questions

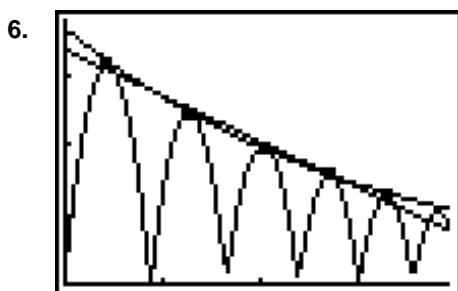
1. The equation of the line of best fit is  $y = -0.66x + 3.35$ .

2. The line fits the data well. However, at some time,  $t$ , the height of the ball will fall below the  $x$ -axis, indicating that the ball has a negative height.

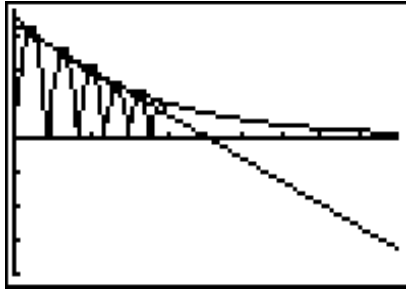


4. The exponential function that best fits this data is  $y = 3.66(0.73)^x$

5. The exponential function is a better fit for this data. This function will model the height of the ball until the ball comes to a complete rest.



7.



8.

Time (seconds)	Rebound Heights	
	linear	exponential
5 s	0.05	0.76
6 s	-0.61	0.55
7 s	-1.27	0.40
8 s	-1.93	0.30
9 s	-2.59	0.22

**Make A Mathematical Statement**

1. The exponential function is the better fit for the data.
2. Over time, the linear model gives negative values for the rebound heights, while the exponential model shows that the rebound heights approach zero.

## Activity 12: Sequence of Bounces

### Instructions

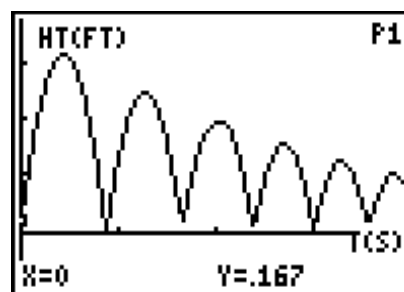
When your students perform this activity, the results will look similar to the practice data below. Calculations in the Questions section are based on the sample data below.

If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1** and **L2**, and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, and the distance in **L2**.

Time (L1)	Dist (L2)
0.00	0.170
0.04	0.710
0.17	2.030
0.34	3.000
0.42	3.140
0.50	3.070
0.71	1.900
0.84	0.490
0.88	0.000
0.92	0.500
1.01	1.350
1.18	2.310
1.26	2.475
1.34	2.380
1.55	1.220
1.60	0.790
1.68	0.170
1.72	0.580
1.93	1.860

Time (L1)	Dist (L2)
2.02	1.959
2.10	1.840
2.31	0.540
2.35	0.130
2.43	0.640
2.60	1.480
2.69	1.562
2.77	1.430
2.94	0.450
2.98	0.080
3.02	0.320
3.19	1.160
3.27	1.249
3.36	1.120
3.48	0.500
3.53	0.200
3.61	0.460
3.74	0.940
3.82	1.020

To view a plot, press **[2nd]** **[STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  $\wedge$  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.

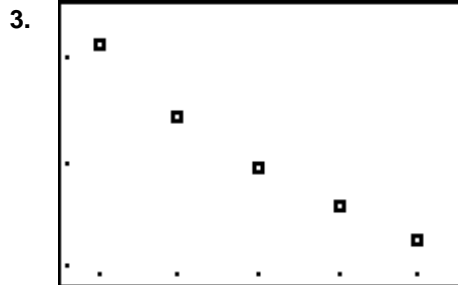


**Data Collection**

1.

$N$	$y$
1	3.140
2	2.475
3	1.959
4	1.562
5	1.249

2. No answer required.

**Questions - Part One**

- $a_1 = 3.14$      $r = 0.794$     This  $r$  is the average of the common ratios found in the table.
- $a_n = 3.14 * (0.794)^{n-1}$   
 $y = 3.14 * (0.794)^{x-1}$
- This sequence models the data well.
- $y = (3.93)(0.794)^x$
- These equations are very close to being the same equation, therefore the new equation fits the data quite well.
- $y = 3.93(0.794)^x \rightarrow y = 3.93(0.794)(0.794)^{x-1} \rightarrow y = 3.12(0.794)^{x-1}$
- The equation in **Y3** should be exactly the same as the equation in **Y2** and very close to the equation in **Y1**.
- The predicted height for the sixth bounce is 0.991 (using the equation in **Y1**) and 0.985 (using the equation in **Y3**).
- The predicted height for the tenth bounce is 0.394 (using the equation in **Y1**) and 0.391 (using the equation in **Y3**).

**Questions - Part Two**

1.	Bounce ( $n$ )	Rebound Height ( $y$ )	Total Distance Traveled for this Bounce	Total Distance Traveled since $t=0$
	1	3.14	$2(3.14) = 6.28$	6.28
	2	2.475	$2(2.475) = 4.95$	$4.95 + 6.28 = 11.23$
	3	1.959	$2(1.959) = 3.918$	$3.918 + 11.23 = 15.148$
	4	1.562	$2(1.562) = 3.124$	$3.124 + 15.148 = 18.272$
	5	1.249	$2(1.249) = 2.498$	$2.498 + 18.272 = 20.77$

2. The total distance should be doubled for every bounce where the bounce begins at the ground and returns to the ground.
3. The sum of the sequence is 10.43. If doubled, it will be 20.86. The distance found in the table is 20.77.
4. No answer required.
5. The sum of the total distance the ball travels for the first six bounces is 22.84.
6. The sum of the total distance the ball travels for the first ten bounces is 27.44.
7.  $2S_n = 2\left(\frac{a_1}{1-r}\right) = \left(\frac{3.14}{1-0.794}\right) = 30.5$



## Activity 13: Good Vibrations

### Notes

- Students should have some knowledge of sinusoidal functions and some experience with geometric transformations. You may want to introduce them to the equation

$$y = A \cos B(x - C) + D.$$

Many books still use the form

$$y = A \cos(Bx + C) + D.$$

This form is more difficult for students to understand than the form used in the activity. The graph is modeled with the cosine function rather than the sine function because the horizontal shift of the cosine is easier to identify than the horizontal shift of the sine curve.

- Be sure that the mass is secure on the spring. Caution students not to pull the spring too far from the equilibrium position. The mass may damage the CBR if it falls on it. You may want to suggest that students practice before placing the CBR beneath the spring and mass.
- If you do not have a stand from which to suspend the spring, you may have a student hold the apparatus. The student should keep his or her arm steady so as not to change the distance from the spring to the CBR.

### Data Collection


When your students perform this activity, the results will look similar to the practice data below. Calculations in the Questions section are based on the sample data below.

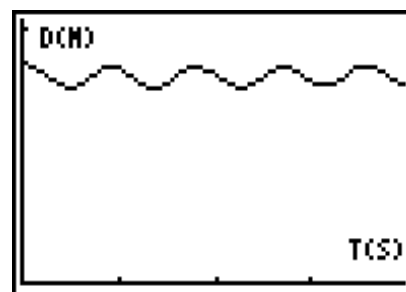
If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1**, **L2**, and **L3** and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, the distance in **L2** and velocity in **L3**. In this example, distances are given in feet rather than meters.

The table below shows the data for the first 2 seconds of the motion, which is enough to perform the calculations. The full set of data is available for download from our web site at [education.ti.com](http://education.ti.com).

L1	L2	L3	L4
0	0.049	-0.077	-0.777
0.04250	0.046	-0.110	-0.885
0.08499	0.042	-0.142	-0.920
0.12749	0.036	-0.170	-0.862
0.16998	0.027	-0.192	-0.706
0.21248	0.017	-0.205	-0.455
0.25498	0.006	-0.207	-0.124
0.29747	-0.006	-0.197	0.261
0.33997	-0.017	-0.174	0.668
0.38246	-0.027	-0.140	1.054
0.42496	-0.035	-0.096	1.379
0.46746	-0.04	-0.044	1.601
0.50995	-0.042	0.011	1.689
0.55245	-0.04	0.065	1.619
0.59494	-0.035	0.116	1.385
0.63744	-0.026	0.157	0.998
0.67994	-0.015	0.184	0.515
0.72243	-0.002	0.194	-0.011
0.76493	0.010	0.185	-0.533
0.80742	0.021	0.158	-1.003
0.84992	0.03	0.117	-1.380
0.89242	0.036	0.065	-1.632
0.93491	0.038	0.006	-1.722
0.9774	0.036	-0.054	-1.650
1.01989	0.031	-0.109	-1.421

L1	L2	L3	L4
1.06238	0.023	-0.153	-1.070
1.10487	0.012	-0.184	-0.622
1.14736	0	-0.197	-0.117
1.18985	-0.013	-0.192	0.398
1.23234	-0.024	-0.17	0.875
1.27483	-0.034	-0.132	1.270
1.31732	-0.040	-0.082	1.549
1.35981	-0.044	-0.024	1.685
1.40230	-0.043	0.035	1.665
1.44479	-0.039	0.091	1.493
1.48728	-0.032	0.138	1.185
1.52977	-0.022	0.173	0.768
1.57226	-0.010	0.191	0.282
1.61475	0.002	0.192	-0.229
1.65724	0.014	0.175	-0.719
1.69973	0.024	0.143	-1.142
1.74222	0.031	0.097	-1.461
1.78471	0.036	0.042	-1.646
1.82720	0.036	-0.017	-1.679
1.86969	0.034	-0.074	-1.557
1.91218	0.027	-0.124	-1.290
1.95467	0.018	-0.163	-0.905
1.99716	0.007	-0.187	-0.438
2.03965	-0.005	-0.193	0.070

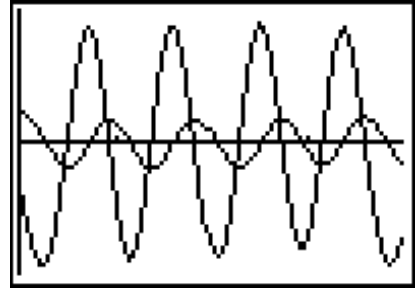
- To view a plot, press **[2nd] [STAT PLOT]** and select **1:PLOT1**. Highlight **On** and press **[ENTER]**. Select  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9:ZoomStat**. The graph should be similar to that shown at the right.



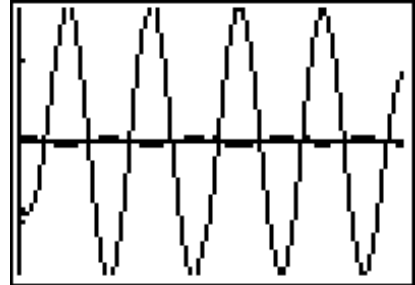
- Equilibrium position is 0.8198. The graph shifts downward when this number is subtracted from **L2**. The window values need to be adjusted to show negative  $y$ -values.
- No answer required.

4.  $A = 0.03796$  which represents the distance that the spring moves from its equilibrium position.
5. Period = 0.8923
6.  $B = 7.042$
7.  $C = 0.9349$
8.  $Y = 0.03796 \cos(7.042(X - 0.9349))$

9. The period of the displacement and velocity are equal. When the distance is at a maximum, the velocity is zero. This makes sense because the mass stops as it turns around to change direction. The mass moves the fastest as it passes through the equilibrium position.



10. The period of the acceleration is the same as the period for the distance plot. When the distance is a maximum, the acceleration is a minimum. The acceleration is zero when the mass passes through the equilibrium position.



11.  $a = -(k/m)x$ . Since  $(k/m)$  is a constant, we see from the graphs that the acceleration graph is the negative of the distance graph \* a constant.

## Activity 14: Swinging Along

### Data Collection

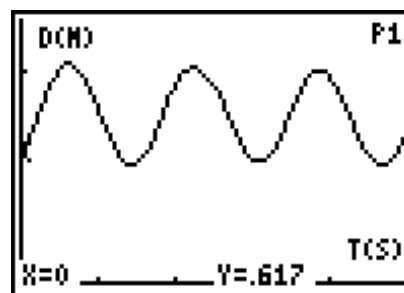
When your students perform this activity, the results will look similar to the practice data below. Calculations in the Questions section are based on the sample data below.

If you would like to practice this activity without collecting the data, you can enter the practice data into lists **L1** and **L2**, and then view a sample plot. To enter the data, press **[STAT]** **[ENTER]**. Type the time in **L1**, and the distance in **L2**.

Time (L1)	Distance (L2)
0	0.62
0.27	0.83
0.53	1.02
0.64	1.03
0.74	1.00
1.01	0.81
1.33	0.57
1.44	0.55
1.54	0.57
1.86	0.80
2.13	0.99
2.23	1.02
2.34	1.01

Time (L1)	Distance (L2)
2.66	0.78
2.98	0.58
3.08	0.57
3.19	0.60
3.45	0.79
3.72	0.98
3.83	1.00
3.93	0.99
4.20	0.82
4.57	0.57
4.68	0.56
4.78	0.59
4.94	0.68

- To view a plot, press **[2nd]** **[STAT PLOT]** and select **1: PLOT1**. Highlight **On** and press **[ENTER]**. Select  $\text{L1}$  for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist** and the **square** for the **Mark**. Press **[ZOOM]** and select **9: ZoomStat**. The graph should be similar to that shown at the right.



- The distance to the CBR to the pendulum's rest position was 0.80 m.
- The distance from the rest position to the point it was pulled back was 0.25 m.
- Three cycles took a total of 4.9 seconds or 1.63 seconds per cycle.

### Questions

- From the graph above, the first maximum is (0.64, 1.03), the minimum is (1.44, .55) and the second maximum is (2.23, 1.02).
- Using the first maximum, the amplitude is 0.24 m. This is within 0.01 m of the distance from the pendulum's rest position to the point it was pulled back.

3. Using the first maximum, the vertical shift is 0.79 m. This is within 0.01 m of the distance from the CBR to the pendulum's rest position.
4. The phase shift is 0.64 m.
5. The period is 1.59 s. This is within 0.04 seconds of the average time.
6. The value of  $B$  is 3.95 s.
7.  $y = 0.24 \cos(3.95(x - 0.64)) + 0.79$
8. The two should be the same.
9. The phase shift for the sine curve is 1.86.
10.  $y = 0.24 \sin(3.95(x - 1.86)) + 0.79$
11. The two should be the same.

